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Soliton equations, Schottky problem and quantum string theory

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The followings are based on the work in collaboration with N. Ishibashi and Y. Matsuo (1986).

Recently Mulase (1984) has shown that every orbit of the Kadmotsev-Petviashvili hierarchy (a class of soliton equations) is locally isomorphic to a cohomology group associated with a commutative ring and that it is a Jacobian variety of an algebraic curve if and only if the orbit is finite dimensional. In this way the soliton equations characterized Jacobians among Abelian varieties (solution to the Schottky problem). The Schottky problem is also becoming important in quantum string theory, where physicists consider quantum field theories on curves. T. Eguchi and the author (1986) have found that certain identities in free fermion systems on curves are implied by addition formulae of theta functions [Fay (1973)], which hold only on Jacobian varieties. Thus it would be natural to expect that there should be direct correspondence between quantum field theories on curves and the theory of soliton equations.

Using the Sato's universal Grassmannian manifold [M. Sato (1981)], we have succeeded in contructing a frame work to treat free fermion systems on curves in a unified manner. We have constructed our formulation in two steps. Firstly, we have coordinatized fermion systems on various curves. The basic mathematical tools needed in this step h ave already been developed by Segal and Wilson (1984). A fermion system is characterized by a set of geometrical data; genus and complex structure of a curve, a line bundle on a curve (fermions are its sections) and its local trivialization. To such geometrical data, we can assign a subspace of $L^2(S^1,\mathbb{C})$, a Hilbert space of square integrable functions, by embeddin S^1 into the curve and considering analytic functions on S^1 which extend outside of it as holomorphic sections of the line bundle. In this way, fermion systems are coordinatized by points in the universal Grassmannian.

The next step is to develop techniques to describe each of these systems; for example to compute correlation functions. In this step we have employed the boson-fermion correspondence, one of the most remarkable aspects of two-dimensional field theories [see for example Alvarez-Gaume, Bost , Moore, Nelson and Vafa (1986) and Eguchi and Ooguri (1986)]. It means that correlation functions of free fermions on a curve are given from those of their chiral current. An insertion of the current along with a real one-dimensional curve on a complex curve changes boundary conditions of fermions across the curve, so it is related to a variation of geometrical data of the fermion system. Accordingly the current insertions is described as a transformation on the universal Grassmannian, and so are correlation functions of free fermions due to the boson-fermion correspondence. These results have been announced in the letter [N. Ishibashi, Y. Matsuo and H. Ooguri (1986)], and the detailed derivations will be given in a separate paper.

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